

یادداشت کنکور، ۱۴۰۵ - ریاضیات
 فردا و ظهر

۱- (۴)

$$A = \frac{(x - \sqrt[3]{2})^3 + 1}{(x - \sqrt[3]{2})^3 + 3} = \frac{(\sqrt[3]{3})^3 + 1}{(\sqrt[3]{2})^3 + 3} = \frac{4}{5} = 0.8$$

۲- (۲)

$$\begin{cases} a_{\varepsilon} = 3a + 2 \\ a_v = 11 \\ a_q = a - 2 \end{cases} \rightarrow \begin{cases} 3d = 9 - 2a \\ 2d = a - 15 \end{cases} \Rightarrow \begin{cases} d = 3 - a \\ d = \frac{a - 15}{2} \end{cases}$$

$$\Rightarrow 3 - a = \frac{a - 15}{2} \Rightarrow a - 15 = 6 - 2a \Rightarrow 3a = 21 \rightarrow a = 7$$

$$\Rightarrow \text{مثبت} : \underbrace{-1, -1, -1, 2, -1, -1, 11, -1, 3, -1, \dots}_{\text{مثبت}}$$

نُه جمله مثبت

۳- (۲)

$$(a-2)^r + 2a(n+1)^r - a = 0$$

$$\Rightarrow (2a+1)x^r + (\varepsilon a - \varepsilon)x + (\varepsilon + a) = 0$$

$$\Delta = 0 \Rightarrow \underbrace{14(a-1)^r - \varepsilon(2a+1)(a+\varepsilon)}_{14a^r - 42a + 14 - 14a^r - 42a - 14} = 0 \Rightarrow \underbrace{14a^r - 71a}_{a(14a - 71)} = 0$$

دو مقدار برابر a وجود دارد.

۲

نبرد اریستو

مسئله (۱)

$$\binom{3}{2} \times \binom{4}{1} \times \binom{4}{1} = 3 \times 4 \times 4 = 27$$

دوره مقادیر دوره یک‌نفر

مسئله (۴)

$$\forall x \in \mathbb{R} : \neg (x \neq 0) \vee (x + \frac{1}{x} \neq 1)$$

$$\xrightarrow{\text{تفین}} \exists x \in \mathbb{R} : \underbrace{(x \neq 0)}_{(x > 0)} \wedge (x + \frac{1}{x} \leq 1)$$

مسئله (۱)

$$\left\{ \begin{array}{l} \alpha^c = \alpha + c \rightarrow \alpha^c = \alpha + c = 7\alpha + 12 \\ \beta^c = \beta + c \rightarrow \beta^c = \beta + c = 7\beta + 9 \end{array} \right.$$

$$\alpha^d, \beta^d = 7\alpha^c + 12\alpha = 7\alpha + 12 + 12\alpha = 19\alpha + 12$$

$$\Rightarrow \alpha^d + \beta^d = 19(\alpha + \beta) + 12 + 12 = 19 + 24 = 2$$

$$\alpha^c - \alpha - c = 0 \rightarrow \alpha + \beta = 1$$

3

مرداد (مدرس)

(3) ج 1

$$y(r) = \varepsilon m - r + \varepsilon = \varepsilon m - r$$

$\downarrow$ $\downarrow$
 a b

$\left\{ \begin{array}{l} y(r) < 0 \\ m > 0 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} \varepsilon m - r < 0 \\ m > 0 \end{array} \right\} \rightarrow m < \frac{1}{\varepsilon} \Rightarrow a + 2b = 1$
 $\left\{ \begin{array}{l} y(r) > 0 \\ m < 0 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} \varepsilon m - r > 0 \\ m < 0 \end{array} \right\} \rightarrow m > \frac{1}{\varepsilon}$

GG'E

(1) ج 1

$$f(m) = \left(\frac{1}{r}\right)^x$$

$$f^{-1}(r-x) \leq f^{-1}(r-n) \xrightarrow{f} f(f^{-1}(r-x)) \geq f(f^{-1}(r-n))$$

$$\Rightarrow r-x \geq r-n \Rightarrow \boxed{n \leq x} \quad (1) \quad \text{نمودار } n=1$$

* جواب: $f^{-1}(m) = \log \frac{1}{\varepsilon} \rightarrow D_{f^{-1}}: n > 0 \Rightarrow \left\{ \begin{array}{l} r-x > 0 \rightarrow n < \varepsilon \\ r-n > 0 \rightarrow n < r \end{array} \right\} \quad (2)$

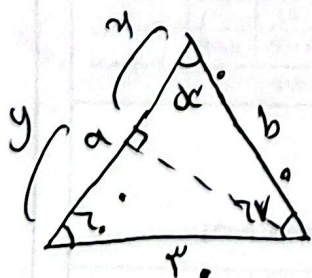
(4) ج 1

$$\sqrt[3]{\delta^3} = \sqrt[3]{r^3} \Rightarrow \sqrt[3]{\delta^3 - r^3} = \delta^2 n$$

$$\Rightarrow (cy-x) \log \frac{r}{\delta} = rx \Rightarrow \frac{x}{cy-x} = \frac{1}{r} \log \frac{r}{\delta}$$

$$\Rightarrow (125) \frac{x}{cy-x} = \left(\frac{r}{\delta}\right)^{\frac{1}{r} \log \frac{r}{\delta}} = \left(\frac{r}{\delta}\right)^{\frac{r}{r} \log \frac{r}{\delta}} = \sqrt[3]{\frac{r}{\delta}}$$

کانون سینوس



(3) ج 1

$$\frac{r}{\sin \delta} = \frac{b}{\sin \gamma} \Rightarrow b = \frac{r \times \frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}r}{\frac{1}{2}} = 2\sqrt{3}r$$

$$a = x + y = b \cos \delta + r \cos \gamma$$

$$= 2\sqrt{3}r \times \frac{1}{2} + r \times \frac{1}{2} = \frac{\sqrt{3}r}{2} + \frac{r}{2}$$

$$\text{مساحت} = \frac{\sqrt{3}r}{2} + \frac{r}{2} + \frac{\sqrt{3}r}{2} + \frac{r}{2} = \varepsilon \delta + \frac{\sqrt{3}r}{2}$$

سوال (۴)

۱-مورد $\begin{cases} \sin \alpha = \frac{3}{5} \\ \cos \alpha = \frac{4}{5} \end{cases}$

۲-مورد $\begin{cases} \sin \beta = \frac{5}{12} \\ \cos \beta = \frac{11}{12} \end{cases}$

$\sin(\alpha + \beta) = \frac{-44 + 30}{70} = \frac{-14}{70} \rightarrow$ (مورد ۲ و ۱) ۳-مورد

$\cos(\alpha + \beta) = 2\cos\alpha\cos\beta - 1 = \frac{2 \times 11 \times 4}{12 \times 5} - 1 > 0 \rightarrow$ (مورد ۲) ۴-مورد

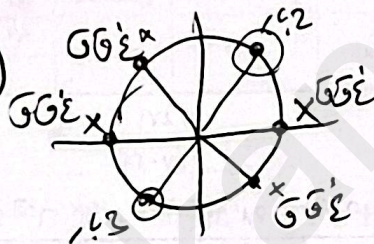
سوال (۱)

$\frac{\sqrt{3} + \tan x}{1 + \sqrt{3} \tan x} = \tan \epsilon x \Rightarrow \tan x = \tan \epsilon x$

$\frac{\tan x + \sqrt{3}}{\tan x}$

$\Rightarrow \epsilon x = x + k\pi \rightarrow x = \frac{k\pi}{3}$

مورد: $(\tan x \neq -\sqrt{3})$



جواب: $-\pi + \frac{\pi}{3} / -\pi + \frac{\pi}{2} / \frac{\pi}{3} / \frac{2\pi}{3}$

سوال (۲)

$\lim_{n \rightarrow \infty} f(g(n)) = 1 + a \Rightarrow f(-a) = 1 + a$

$a \in \mathbb{Z} : [1 - a] = 1 - a$

$\frac{a^+}{0} [1 - a] = (1 - a) - 1 = -a$

$1 - \sqrt{-a} = 1 + a$

$\Rightarrow -a = \sqrt{-a}$

$\xrightarrow{\text{مورد ۲}} a^+ = -a \begin{cases} a = 0 \\ a = -1 \end{cases}$

جواب:

در ادامه

$$x = -1+t \rightarrow \lim_{t \rightarrow 0} \frac{\cos(-2\pi + 2\pi t) - 1}{-1+t+2\sqrt{1-t}-1}$$

(۲) ۱۴

$$= \lim_{t \rightarrow 0} \frac{\cos 2\pi t - 1}{-(\sqrt{1-t}-1)^2} = \lim_{t \rightarrow 0} \frac{2 \sin^2 \pi t}{(\sqrt{1-t}-1)^2}$$

$$\xrightarrow{\text{HOP}} \frac{\epsilon \pi \sin \pi t \cos \pi t}{2(\sqrt{1-t}-1) \left(\frac{-1}{2\sqrt{1-t}}\right)} = -\frac{\epsilon \pi \sin \pi t}{\sqrt{1-t}-1} \xrightarrow{\text{HOP}} \frac{-\epsilon \pi^2 \cos \pi t}{\frac{-1}{2\sqrt{1-t}}} = \underline{1\pi^2}$$

$$\lim_{n \rightarrow \frac{1}{r}} \frac{1-2n}{2n^r + n + a}$$

(۳) ۱۵

$$a \rightarrow \frac{1}{r} \quad 2n^r + n + a \Rightarrow \frac{2}{r} + \frac{1}{r} + a = a + r = 0 \rightarrow a = -r$$

$$\therefore \lim_{n \rightarrow \frac{1}{r}} \frac{1-2n}{2n^r + n + a} \times \frac{1-2n}{(2n^r + n + a)} = \frac{1 + \frac{\epsilon}{2}}{\frac{\epsilon}{2} \times \left(\frac{-r}{2}\right)} = \underline{\frac{2}{\epsilon}}$$

$$f(n) = a(1-n^r) - b n^r = a - \underbrace{(a+b)}_{\text{درجه ۰}} n^r \Rightarrow b = -a$$

(۱) ۱۶

$$\lim_{n \rightarrow a^-} \frac{n \times a}{n - a} = \frac{a^r}{0^-} = -\infty$$

✓

لبردار (پارامتر)

$$\begin{aligned}
 f(1) &= 0 \rightarrow a+b+c-d=0 \xrightarrow{c=0} \boxed{a+b=d} \quad (1) \\
 f'(0) &= 0 \rightarrow \left. \frac{d}{dx}(ax^2+bx+c) \right|_{x=0} = 0 \rightarrow \boxed{c=0} \\
 f'(-1) &= 0 \rightarrow c-a-b=0 \rightarrow \boxed{c=a=b} \\
 \begin{cases} a+b=d \\ c=a=b \end{cases} &\Rightarrow a=b, c=b \Rightarrow a=b, c=b
 \end{aligned}$$

$$\begin{aligned}
 f(-1) &= -a+b-d \\
 &= -b+b-d = -d
 \end{aligned}$$

خط فقر = نصف میانگین درآمد به ازای هر نفر

$$\frac{1}{2} \left(\frac{21+21+22+24+25+27+29}{7} \right) = \frac{178}{14} = 12.71$$

$$P(A) = \frac{\binom{k}{k-d}}{2^k - 2} \xrightarrow{k=7} \frac{\binom{7}{6}}{2^7 - 2} = \frac{7}{126} = \frac{1}{18}$$

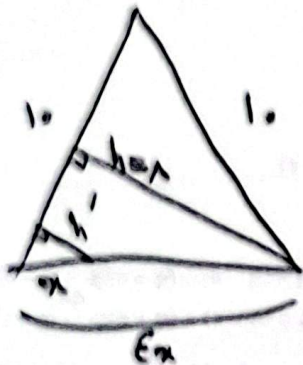
7

مردا و ملازمت

(۲۵) ک

$$P(A) = \frac{\binom{6}{3} \times 1 \times 1 \times 1 \times 1 \times 1 \times 1}{6^6} = \frac{1 \times 17}{6^6} = \frac{17}{216}$$

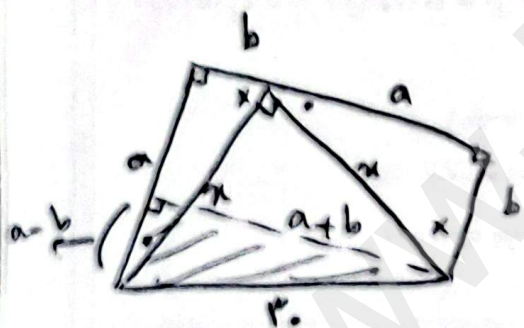
(۲۶) ک



$$\frac{h \times 1}{2} = 1 \rightarrow h = 2$$

$$\frac{h'}{h} = \frac{1}{2} \rightarrow h' = \frac{1}{2} \times 2 = 1$$

(۲۷) ک

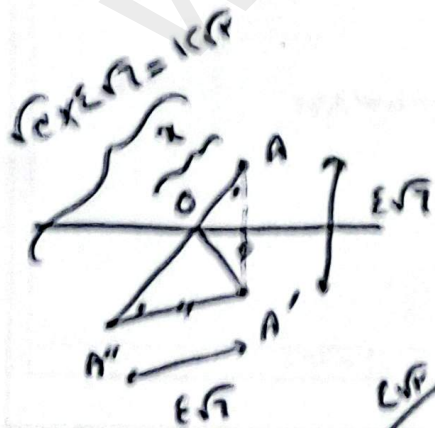


$$a^2 + b^2 = c^2 \rightarrow a = 15\sqrt{2}$$

$$a^2 + b^2 = c^2 = 900 = 30^2$$

$$\begin{cases} a, b \in \mathbb{Z} \\ a + b \in \mathbb{Z} \end{cases} \rightarrow \begin{cases} a = 15, b = 15 \\ a + b = 30 \end{cases}$$

(۲۸) ک



$$OAA' \sim AA'A''$$

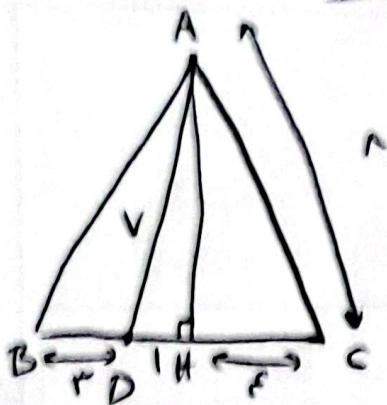
$$\Rightarrow x \times \sqrt{2} = \sqrt{2} \times \sqrt{2}$$

$$x = \frac{\sqrt{2}}{\sqrt{2}} = 1 = \sqrt{2}$$

$$\sqrt{2^2 - 1^2} = \sqrt{3} = \sqrt{2}$$

$$S_{OAA'} = \frac{1}{2} \times \sqrt{2} \times \sqrt{2} = 1$$

اسرار و فلسفہ



$$AH = \frac{\sqrt{r}}{f} \times h = \varepsilon \sqrt{r}$$

۲۹ (۳)

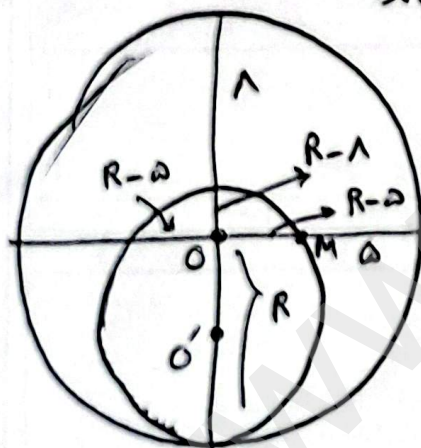
$$AD' = AH' + HD' \Rightarrow v' = \varepsilon h + HD'$$

$$\Rightarrow HD = 1$$

$$BD = r$$

۳۰ (۳)

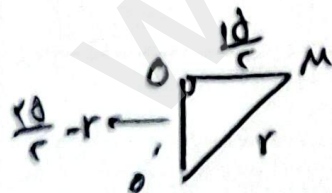
ر و د و ب و ج



$$R(R-\lambda) = (R-\delta)r$$

$$\Rightarrow R^2 - \lambda R = R^2 - 1 \cdot R + r\delta$$

$$\Rightarrow rR = r\delta \rightarrow (R = \frac{r\delta}{r})$$



$$\Rightarrow r' = (\frac{r\delta}{r} - r)' + (\frac{1}{\delta})r$$

$$r' = r' + \frac{r\delta}{\varepsilon} - r\delta r + \frac{r\delta}{\varepsilon}$$

$$\Rightarrow r = \frac{\lambda \delta \cdot}{\varepsilon \times \delta} = \lambda_1 \delta$$

$$OO' = R - r = 1 \cdot \delta - \lambda_1 \delta = \varepsilon$$

$$\vec{a} \times \vec{i} = \begin{pmatrix} 1, -1, -1 \\ 1, 0, 0 \end{pmatrix}$$

$$\underline{\underline{(0, -1, +1)}}$$

$$\vec{k} \times \vec{a} = \begin{pmatrix} 0, 0, 1 \\ 1, -1, -1 \end{pmatrix}$$

$$\underline{\underline{(+1, +1, 0)}}$$

دکتر (1)

$$\vec{k} \times (\vec{a} \times \vec{i}) + (\vec{k} \times \vec{a}) \times \vec{i} = (1, 0, 0) + (0, 0, -1)$$

$$\begin{pmatrix} 0, 0, 1 \\ 0, -1, 1 \end{pmatrix} \quad \begin{pmatrix} 1, 1, 0 \\ 1, 0, 0 \end{pmatrix} = \vec{i} - \vec{k}$$

$$A = \begin{bmatrix} -2 & y-5 \\ -\varepsilon\delta - c\alpha & \sqrt{c} + \pi y \end{bmatrix} \rightarrow \begin{cases} y=5 \\ \alpha=-15 \end{cases} \rightarrow \frac{\pi}{y} = -3$$

دکتر (6)

$$(AB)^{-1} A^T B = B^{-1} A^T B = \begin{bmatrix} \delta |A| & \varepsilon \\ |A| & \gamma |A| \end{bmatrix}$$

$$B^{-1} A^T A^T B$$

$$\rightarrow |B^{-1}| |A^T| |B| = 1 \cdot |A|^T - \varepsilon |A|$$

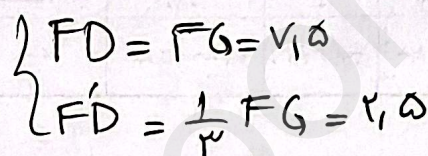
$$\Rightarrow 9 |A|^T = \varepsilon |A| \quad \begin{cases} |A| = 0 \\ |A| = \frac{\varepsilon}{9} \end{cases}$$

$$|cA| = 9 |A| = \varepsilon$$

دکتر (1)

11

$$\sigma_4 \text{ da } \varepsilon_y - \omega = \frac{\varepsilon_y}{\omega} \rightarrow r = \frac{1}{\sqrt{14+9}} = \frac{1}{5} = 0.2$$



$$Y_C = FF' = 9 \rightarrow \epsilon = \epsilon_d$$

$$e = \frac{c}{a} = \frac{\Sigma d}{a} = \underline{1.91}$$

$\Sigma A = 1 \times 7 = 7$

$$\frac{17}{2} = \frac{187 + 21k}{2} = 94 + 1k \hat{=} 94 + 1k$$

(۱) ۷-۲

$$7x + 12y = 297 \Rightarrow xy \equiv 3 \pmod{10} \Rightarrow y \equiv 7$$

$$y = 7k + 7$$

$$\begin{cases} y = 7k + 7 \\ x = -12k + 29 \end{cases} \xrightarrow{-5x \leq 145} k = 2 \rightarrow \begin{cases} y = 19 \\ x = 15 \end{cases} \xrightarrow{+} (1)$$

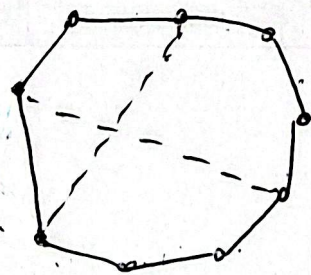
$$0 \leq 7 \rightarrow \begin{cases} \text{---} 4 \Rightarrow 1 \\ \text{---} 4 \Rightarrow 1 \\ \text{---} 4 \Rightarrow 1 \end{cases} \xrightarrow{+} 1 + 1 + 4 = 15$$

$$+ \rightarrow 1 + 1 + 4 = 15$$

$$n = 4$$

(۱) ۷-۲

(۳) ۷-۲



دور

دور به طول ۳ → ۲

دور به طول ۲ → ۲

دور به طول ۱ → ۱

دور به طول ۱ → ۱